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Density of states and tachyons in open and closed string theory

Vasilis Niarchos

Enrico Fermi Institute, and
Deptartment of Physics, University of Chicago
5640 S. Ellis Ave., Chicago, IL 60637, USA
E-mail: vniarcho@theory.uchicago.edu

Abstract: In this note we reexamine the possibility of constructing stable non-supersymmetric theories that exhibit an exponential density of states. For weakly coupled closed strings there is a general theorem, according to which stable theories with an exponential density of states must exhibit an almost exact cancellation of spacetime bosons and fermions (not necessarily level by level). We extend this result to open strings by showing that if the above cancellation between bosons and fermions does not occur, the open strings do not decouple from a closed string tachyon even in the NCOS scaling limit. We conclude with a brief comment on the proposed generalization of the AdS/CFT correspondence to non-supersymmetric theories.

Keywords: D-branes, Superstring Vacua, Supersymmetry Breaking.

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1. Introduction

In field theory one can pose the following question. Is it possible to find stable, non-supersymmetric, weakly interacting theories with an exponential density of states? This is an interesting question for a number of reasons. For example, QCD appears to be an example of this type of theories. In the limit of a large number of colors it contains an infinite number of weakly interacting mesons [1, 2]. These states are expected to lie on Regge trajectories and their density to be exponential. A different reason is the more general question of supersymmetry breaking in string theory and in particular the problem of the cosmological constant.

String theories generically possess a Hagedorn spectrum of states and therefore offer a suitable arena to discuss the above questions. The situation for closed string theories has been analyzed in [3, 4] (see also [5, 6] for a related discussion) and has been found that in the context of a generic weakly coupled closed string theory the existence of tachyon instabilities and the density of states are related. More precisely, whenever a string theory is tachyon free, the number of bosons almost cancels the number of fermions. Following [3, 4] we refer to this almost exact cancellation by using the term *asymptotic supersymmetry.* A special case of this phenomenon occurs for supersymmetric theories, because supersymmetry guarantees the exact matching of the number of bosons and fermions in the physical spectrum. In general, however, spacetime supersymmetry is not necessary for a stable theory with an exponential density of states and this matching may or may not be strictly exact. The necessary property is asymptotic supersymmetry. We review the relevant arguments in section 2.

Along these lines, it is natural to ask whether a similar situation appears in open string theories as well. Naively, this does not seem to be the case, because there are known examples of string theories with open string spectra that include an exponential density of states without the above boson-fermion cancellation and without an open string tachyon. As we are going to see, however, an open string spectrum with an exponential density of states and no asymptotic supersymmetry cannot be IR stable in the weak coupling regime, because it cannot be decoupled from a *closed* string tachyon even in the NCOS (non-commutative open string) scaling limit. Hence, asymptotic supersymmetry is needed in open string theories as a condition for stability of the bulk and not as a condition for stability of the branes.

As an example of this general phenomenon, we discuss D-branes in the type-0 theories and comment on the proposed AdS/CFT correspondence in this class of theories.

2. Closed string theory

We mentioned above that in weakly coupled closed string theory, there is a general theorem relating the existence of tachyon instabilities with the density of states. Following [3, 4] let us briefly review this result. Consider a generic closed string vacuum. The torus partition function takes the general form

$$Z_T(\tau) = \mathrm{Tr}_{\mathcal{H}_c}\, q^{L_0 - \frac{c}{24}} \overline{q}^{\overline{L}_0 - \frac{c}{24}}\,, \tag{2.1}$$

where $q = e^{2\pi i\tau}$ and $\tau = \tau_1 + i\tau_2$ is the worldsheet modulus. In terms of this partition function the one-loop free energy Ω equals

$$\Omega = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau)\,, \tag{2.2}$$

where $Z(\tau) \equiv \tau_2 Z_T(\tau)$ and $\mathcal{F}$ is the fundamental domain of the moduli space of the torus. In field theory the corresponding amplitude has a UV divergence and needs to be regularized, but in closed string theory this divergence is cut off automatically by restricting the integral to the fundamental domain. Thus, the only possible divergence of Ω is an IR one (coming from $\tau_2 \to \infty$) and is associated with the presence of tachyons in the physical spectrum of the theory.

Now define the following function

$$G(\tau_2) \equiv \int_{-1/2}^{1/2} d\tau_1 Z(\tau)\,. \tag{2.3}$$

The integration over τ_1 correctly implements the level matching condition $L_0 = \overline{L}_0$ for the left and right movers. In terms of the Coleman-Weinberg formula, which gives Ω as

$$\Omega = \mathrm{Tr}(-1)^F \log(p^2 + m^2) = \int_0^\infty \frac{ds}{s} \sum_n (-1)^{F_n} \int d^D p e^{-s(p^2 + m_n^2)} \tag{2.4}$$

the G function reads

$$G(\tau_2) = \tau_2 \sum_n (-1)^{F_n} \int d^D p \; e^{-2\pi\tau_2(p^2+m_n^2)} \,. \qquad (2.5)$$

From this formula we see in particular that spacetime fermions contribute with a minus sign and therefore it is natural to write G as

$$G = G_B - G_F \,. \qquad (2.6)$$

In the $\tau_2 \to 0$ limit both G_B and G_F take the general form

$$G_{B,F} \simeq \tau_2^x e^{y/\tau_2} \qquad (2.7)$$

and can be thought of as regularized versions of the number of states of the system. In particular, G can be thought of as a measure of the difference between the number of bosonic and fermionic degrees of freedom in the physical spectrum of the theory.

The theorem of references [3, 4] postulates that under very mild assumptions the following relation holds

$$\Omega = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau) = \frac{\pi}{3} \lim_{\tau_2 \to 0} G(\tau_2) \,. \qquad (2.8)$$

In any infrared stable theory the l.h.s. of this equation is finite. It follows then that

$$\lim_{\tau_2 \to 0} G(\tau_2) = \lim_{\tau_2 \to 0} (G_B(\tau_2) - G_F(\tau_2)) = \text{const} \,. \qquad (2.9)$$

Hence, in any tachyon-free theory bosons and fermions in the physical spectrum almost cancel. This cancellation takes place in the asymptotic high energy limit $\tau_2 \to 0$ and may not be exact or level by level, but the total difference of states up to some high energy level must asymptotically approach the low residual density of states counted by G. In fact, as explained in [3, 4], the G function converging to a constant in the $\tau_2 \to 0$ limit corresponds to a 2-dimensional field theoretic density of states. This phenomenon of asymptotic cancellation between the number of bosons and fermions up to a 2-dimensional density of states is what we refer to as asymptotic supersymmetry. The conclusion is that any stable weakly coupled closed string theory, if not supersymmetric, has to be at least asymptotically supersymmetric.

From this point of view closed string tachyon condensation proceeds in the following way. Suppose we start with an unstable weakly coupled closed string theory. Then, the spectrum of this theory does not have asymptotic supersymmetry and includes for example an exponential density of purely bosonic degrees of freedom. After tachyon condensation there are two possibilities. Either the new theory is strongly coupled, in which case we cannot say anything from the point of view of our analysis, or the new theory is still weakly coupled. In that case, the new vacuum

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must be asymptotically supersymmetric. That means that either an exponential density of fermions has dynamically appeared to match asymptotically the density of bosons, or more drastically spacetime collapses down to two dimensions by some violent mechanism (probably of the sort encountered in [7]). This qualitative picture also seems to be in agreement with the picture one has about closed string tachyon condensation from the point of view of the worldsheet. From that point of view the closed string tachyon corresponds to a relevant perturbation on the worldsheet and under the renormalization group the worldsheet theory flows from the UV to the IR. Because of the c-theorem, the c-function will have a smaller value in the IR and this suggests either that fermionic degrees of freedom have appeared in the new stable vacuum, or that the spacetime dimension has been drastically reduced.

3. Open string theory

For closed string theory modular invariance was critical to the proof of the above theorem. For open strings analogous results can be obtained by using worldsheet duality. Open string theory generally possesses different sectors, but for the moment let us restrict our discussion to the case of an ab sector, corresponding to boundary conditions $|a\rangle, |b\rangle$. For this sector we consider the annulus partition sum

$$\Omega_{ab} = \int_0^\infty \frac{dt}{t} \operatorname{Tr}_{\mathcal{H}_{ab}} q^{L_0-\frac{c}{24}} = \int_0^\infty \frac{dt}{t} A_{ab}(t) \,, \tag{3.1}$$

where q is now redefined to be $q = e^{-2\pi t}$ and $A_{ab}(t) \equiv \operatorname{Tr}_{\mathcal{H}_{ab}} q^{L_0-\frac{c}{24}}$. Again, the Coleman-Weinberg formula (2.4) gives

$$A_{ab}(t) = \sum_{n\in ab} (-1)^F \int d^D p \; e^{-2\pi t(p^2+m_n^2)} \,, \tag{3.2}$$

where by $\sum_{n\in ab}$ we signify a sum over the open string states of the ab sector. In parallel to the closed string case, the $t\to 0$ limit of $A_{ab} = A_{B,ab} - A_{F,ab}$ counts the asymptotic difference between the number of bosonic and fermionic degrees of freedom within the ab sector and in general both $A_{B,ab}$ and $A_{F,ab}$ take (in this limit) the form (2.7)

$$A_{B,F}(t) \simeq t^x e^{y/t} \,. \tag{3.3}$$

As we know [8, 9], we may use worldsheet duality to rewrite A_{ab} in terms of the transverse closed string channel variable $\tilde{q} = e^{-\pi/t}$

$$A_{ab}(t) = 2t \langle b| \tilde{q}^{\frac{1}{2}\left(L_0+\overline{L}_0-\frac{c}{12}\right)} |a\rangle \tag{3.4}$$

and reinterpret the above UV divergences (from the $t\to 0$ limit) as IR effects associated to the propagation of closed string modes. An exponential divergence signals the existence of a closed string tachyon with a non-zero coupling to the open strings. This tachyon has a negative mass squared $-2y/\alpha'$ in direct relation to the power of the exponential divergence in (3.3). Similarly, the next dominant term (or the first dominant term if there are no tachyons in the bulk) is a negative power that corresponds to the propagation of massless closed string modes. This power

$$A_{ab}(t) \simeq t^x \tag{3.5}$$

can be determined in the following way. For momentum k^μ flowing between the cylinder boundaries, the annulus amplitude (3.1) also includes a factor $e^{-\alpha' k^2 s/2}$ in the large $s = 1/t$ limit. Of course, without any external open string insertions this momentum is zero, but we can insert it as a regulator into the annulus partition sum

$$\Omega_{ab} \simeq \int_0^\infty \frac{dt}{t} t^x e^{-\omega/t} \tag{3.6}$$

and afterwards take the limit $\omega = \frac{1}{2}\alpha' k^2 \to 0^+$. After a change of variables this amplitude becomes

$$\Omega_{ab} \simeq \int_0^\infty \frac{ds}{s} s^{-x} e^{-\omega s} = \omega^x \int_0^\infty dz z^{-x-1} e^{-z} = \left(\frac{1}{2}\alpha' k^2\right)^x \Gamma(-x)\,. \tag{3.7}$$

From the field theory point of view this result should be proportional to the momentum pole $\frac{1}{k^2}|_{k^\mu=0}$. Clearly, this can only happen when $x = -1$, in which case, as we take the limit $\Lambda^{-2} \simeq t \to 0$, $A_{ab}(\Lambda) \simeq \Lambda^2$ and therefore corresponds to a 2-dimensional field theoretic density of states.

In summary, we conclude that in any given open string sector a net exponential density of states yields a non-zero coupling of the open strings to a closed string tachyon. On the other hand, the absence of this coupling to a closed string tachyon implies a cancellation between the bosonic and fermionic degrees of freedom up to a 2-dimensional density of states. This result is analogous to the one that has been obtained in the closed string case, but there are also some major differences. In the closed string case the stability of the theory was synonymous to asymptotic supersymmetry, but in the open string case instabilities can be due to either an open or a closed string tachyon. It is the absence of a coupling to a closed string tachyon that is equivalent with asymptotic supersymmetry. Open string tachyons, instead, signal the existence of lower energy configurations for the D-branes in question. This observation seems to be implying something deep about the importance of asymptotic supersymmetry in weakly coupled string theories. Its presence, even for those degrees of freedom that live on the branes, appears to be intimately connected to the nature of spacetime. This phenomenon has also been observed in matrix theory, where asymptotic supersymmetry seems to be a crucial requirement for locality and cluster decomposition [10].

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The same arguments can be applied beyond the spectrum of a particular open string sector ab. To implement our conclusions for a collection of open sectors we would have to consider the annulus partition sum

$$\Omega_A = \sum_{ab} \Omega_{ab} = \int \frac{dt}{t} A(t)\,. \tag{3.8}$$

where $A(t) \equiv \sum_{ab} A_{ab}(t)$ and the summation is performed over all the sectors we are considering. Apart from this minor modification the preceding discussion goes through unchanged.

The non-commutative open string case It has been argued in a recent series of papers [11, 12], that there are consistent open string theories without a coupling to the closed sector. If a full decoupling of this sort were always possible, tachyon free open string theories with an exponential density of states and no asymptotic supersymmetry would be examples of the stable theories discussed in the introduction. But these are precisely the kind of string theories we excluded by the above considerations. The resolution to this problem is rather straightforward. A careful examination of the decoupling argument in the NCOS shows that the closed string tachyon and a 2-dimensional portion of the massless closed string spectrum do not in general decouple from the open sector.

The idea behind the NCOS construction is to start with an ordinary string theory of open plus closed strings and take a particular scaling limit by turning on a near critical electric field. The resulting open string theory has the same spectrum and the same annulus partition sum as before but it is described by a new effective string length given by the fixed value $\alpha'_{\text{eff}} = \frac{\theta}{2\pi}$, θ being the noncommutativity parameter, a new open string coupling G_o and a fixed open string metric G_{MN}, which can be set to η_{MN}. At the same time, the closed string coupling g_s is scaled to infinity, whereas the closed string metric is given by the form $g_c = \begin{pmatrix} g & 0 \\ 0 & -g \end{pmatrix} \otimes 1_{(D-2)\times(D-2)}$ and g is also sent to infinity. For simplicity, we have assumed that the branes have only one common direction and that the near critical electric field is turned on in this direction, which we set as the direction 1. In this context, consider the process of an open string turning into a closed string. The open string dispersion relation reads

$$(p^0)^2 = (p^1)^2 + m_o^2 \tag{3.9}$$

and the closed string dispersion relation

$$\frac{1}{g}(p^0)^2 = \frac{1}{g}(p^1)^2 + (\vec{p})^2 + m_c^2\,. \tag{3.10}$$

For $g \to \infty$ we get

$$0 = (\vec{p})^2 + m_c^2\,. \tag{3.11}$$

If $m_c^2 > 0$, the above relation cannot be satisfied and the respective closed string modes decouple, as in the original argument. This is not the case, however, for $m_c^2 \leq 0$. In particular, the tachyon has $m_c^2 < 0$ and therefore does not decouple by the above kinematic argument. Hence, even in the NCOS scaling limit, an open string theory with an exponential density of states and no asymptotic supersymmetry cannot avoid coupling to a closed string tachyon and therefore must be IR unstable. Furthermore, for $m_c^2 = 0$, the massless modes too do not completely decouple. Kinematically, the modes that remain coupled to the open sector are confined in the direction of the electric field and become effectively 2-dimensional degrees of freedom. Of course, gravity decouples completely, since in our limit $\alpha' \to 0$ and gravity always decouples at low energy.

An example. As a further illustration, we would like to present an example. For our purposes, a convenient setup is provided by the type-0 theories. The D-brane spectrum of these theories has been analyzed and has been found to consist of two types of mutually consistent branes plus their anti-branes.[1] Following [14, 15] the first type, which will be called electric, is given by the boundary state

$$|Dp; +, +\rangle = |Dp, +\rangle_{\mathrm{NSNS}} + |Dp, +\rangle_{\mathrm{RR}} \tag{3.12}$$

whereas the second type, which will be called magnetic, is given by

$$|Dp; -, -\rangle = |Dp, -\rangle_{\mathrm{NSNS}} + |Dp, -\rangle_{\mathrm{RR}} \,. \tag{3.13}$$

The $+$,$-$ signs in this notation refer to the choice of different spin structures one can use to define the gluing conditions satisfied by these boundary states. A detailed construction can be found in [14].

The open string spectrum of these branes is the following. Strings stretching between branes of the same type belong to the NS^+ sector, whereas strings stretching between branes of different type belong to the R^+ sector. For this theory a generic configuration of parallel D-branes exhibits a tachyon-free open string spectrum with an exponential density of states and no asymptotic supersymmetry. As one might expect, these branes also have a non-trivial coupling to a closed string tachyon (see references [16, 17]) and this coupling will also persist in the NCOS scaling limit by virtue of the above discussion.

In addition, it has been noted in [16, 17, 18] that the above coupling to the bulk tachyon vanishes for a dyonic pair consisting of an electric plus a magnetic brane. From our point of view, this is clearly expected. On a dyonic pair of branes the open string spectrum is asymptotically supersymmetric and the closed string tachyon decouples. In fact, it is worth noticing that in this case the number of bosons exactly matches the number of fermions level by level, although there is no spacetime supersymmetry [19]. In [16, 17] it was argued that for weak t' Hooft coupling such

[1] Open string descendants of the 0B type were constructed by orientifold projection in [13].

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a D-brane system exhibits a closed string tachyon stabilization and that AdS/CFT correspondence in this non-supersymmetric setting makes sense. In that argument, however, tachyon condensation is a tree-level effect at weak coupling and the bulk spectrum continues to have no asymptotic supersymmetry. Thus, it is not clear how such a mechanism could be reconciled with the analysis of section 2. This difficulty with the tachyon is even stronger for a stack of N branes of the same type. The closed string tachyon in the bulk cannot be stabilized in the weak coupling regime because it must remain coupled to the $\mathrm{U}(N)$ gauge theory on the branes. Therefore, in such a generalization of the AdS/CFT correspondence to non-supersymmetric theories one cannot neglect the closed string tachyon on the "CFT" side.

4. Conclusions

In this note we posed the following question. Is it possible to find stable, non-supersymmetric, weakly interacting theories with an exponential density of states? Probing this question in the context of perturbative string theory, we commented on the correlation between tachyon instabilities and exponential densities of states without asymptotic supersymmetry. We saw that, as in the case of closed string theories, the density of states of an open string spectrum dictates the coupling of the open strings to closed string tachyons. It was stressed that this coupling is not an accidental fact that can be removed by decoupling the closed sector. Even in the NCOS scaling limit this coupling remained as needed by the above discussion on the density of the open string states.

A natural question to ask is the following. What is the meaning of these conclusions for large-N QCD? Clearly, there are three possibilities. One possibility is that large-N QCD presents an example of a stable, weakly coupled theory with an exponential density of states. In that case, however, the discussion of this note shows that it cannot be described by a weakly coupled string theory of the usual kind and one must find a different kind of string theory to describe it. Another alternative is that large-N QCD does not present an example of the above type of theories, i.e. either it does not have an exponential density of states or it is not weakly coupled.

Finally, in recent years some evidence appeared for a deep connection between asymptotic supersymmetry and the emergence of large spacetimes with approximately local physics in string theory (see e.g. [10] or for a recent discussion [20]). The analysis in this note is compatible with these ideas. Indeed, consider a vacuum of string theory which contains D-branes. If the spectrum of states that live on the D-branes is not asymptotically supersymmetric, the vacuum must suffer from a closed string tachyon instability, and spacetime (even very far from the brane) will collapse in the sense that was described at the end of section 2. This seemingly counter-intuitive relation between large breaking of supersymmetry on a brane, and instability of the bulk of spacetime might be a further sign of the above connection.

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